Stats: Critical Region For Binomial Hypothesis Testing Notes

If a result is in the critical region, we reject H_0; if not, we do not reject H_0.

E1: Ase buys sweets with her lunch with probability 0.27. Over the next 20 days, she buys sweets with her lunch once. Find the critical region and test the claim that Ase now buys fewer sweets with her lunch at the 5% significance level.

Method	**Working**
1) Define p:	Let p be the probability Ase buys sweets with her lunch
2) Give the distribution:	$X \sim B(20, 0.27)$
3) State the hypotheses:	$H_0: p = 0.27$ $H_1: p < 0.27$
4) Use a calculator to find cumulative probabilities:	See below

Casio fx-991EX Classwiz	**Casio fx-CG 50**
1) Press MENU and select 7:Distribution	1) Press MENU and select 2 Statistics
2) Press DOWN and select 1:Binomial CD	2) Input the required numbers into List 2*
3) Select 1:List	3) Input 0's in List 1 to match the values in List 2
4) Input the required numbers into the x column*	4) Press F5 for DIST and F5 again for Binomial
5) Press = with any x value highlighted	5) Press F2 for Bcd and F1 for List
6) Input the appropriate N and p values	6) Select L.List, press F1, type 1, press EXE
7) Press =	7) Select U.List, press F1, type 2, press EXE
8) The probabilities we need are in the p column	8) Input the appropriate Numtrial and p values
	9) Select Save Res, press F2, type 3, press EXE
	10) Press EXE again and then press EXIT twice
	11) The probabilities we need are in List 3

*To find the required numbers, find the mean (np). Then:

- If H_1 is $p < k$, we want all the whole numbers from 0 up to the mean
- If H_1 is $p > k$, we want all the whole numbers from the mean up to n
- If H_1 is $p \neq k$, we **may** want all the whole numbers from 0 to n

The table shows how to find $P(X \leq x)$ for all the x-values in the x column (Classwiz) or List 2 (CG 50).

5a) Give the highest probability below the significance level:	$P(X \leq 1) = 0.0155$
5b) Write 'BUT':	BUT
5c) Give the first probability above the significance level:	$P(X \leq 2) = 0.0635$
6) Give the critical region [5a without P and brackets]:	$X \leq 1$
7) Test the result in the question:	$1 \leq 1$ – it's true!
8) If true, reject H_0; if not, do not reject H_0:	Reject H_0
9) Give the conclusion in context:	There is some evidence to suggest that the probability Ase buys sweets with her lunch has reduced

With thanks to Bella Woodcock.

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E2: Mike gets Maths questions right with probability 0.81. He revises thoroughly before the next Maths test and gets 15 of the 16 questions right. Mike claims he has increased his probability of answering Maths questions correctly. By finding the critical region, test his claim at the 10% significance level.

Method	**Working**
1) Define p:	Let p be the probability that Mike gets a Maths question correct
2) Give the distribution:	$X \sim B(16, 0.81)$
3) State the hypotheses:	H_0: $p = 0.81$ H_1: $p > 0.81$
4) Use a calculator to find cumulative probabilities:	$16 \times 0.81 = 12.96$; need 13 up to 16
5a) Give the highest probability below the significance level:	$1 - P(X \leq 15) = 0.0344$ So $P(X \geq 14) = 0.0344$
5b) Write 'BUT':	BUT
5c) Give the first probability above the significance level:	$1 - P(X \leq 14) = 0.163$ So $P(X \geq 15) = 0.163$
6) Give the critical region:	$X \geq 14$
7) Test the result in the question:	$15 \geq 14$ – it's true!
8) If true, reject H_0; if not, do not reject H_0:	Reject H_0
9) Give the conclusion in context:	There is some evidence to suggest that the probability that Mike gets Maths questions correct has increased

Notes For The Above Question

- If H_1 is $p > k$, the critical region takes the form $X \geq x$. As the Classwiz gives us $P(X \leq x)$ only, we must subtract results from 1 to get the required probabilities
- CG 50 owners can change List 1 to the values indicated in Step 4, and List 2 to all n values (16, here) to match the values in List 1. This finds the required probabilities without needing to subtract from 1

General Critical Region Notes

- The set of values that mean we do not reject H_0 is called the **acceptance region**
- We can give critical regions as sets of numbers. E1's is {0, 1}; E2's is {14, 15, 16}
- A two-tailed test (where H_1 is $p \neq k$) has small and large values in it: check **both** sides of the mean

SBNB Only find the critical region if it is required. Use the probability finding method otherwise!

With thanks to Bella Woodcock.